Mr. G. Beber,
Wailuku, Maui
Territory of Hawaii,
U.S.A.

Dear Mr. Beber,

I enclose the reprint you asked for.

F region Z echoes have been observed here at night with a Z critical frequency down to about 1.5 Mc/s. However, at this frequency it is very difficult on a P′f record to distinguish the Z trace from the X trace which shows a critical frequency at the gyro frequency.

I would be interested to hear if you observe triple splitting in Hawaii. At your relatively low geomagnetic latitude the F region would have to be unusually rough for triple splitting to occur by the mechanism described in the enclosed paper.

Yours sincerely,

(G. R. Ellis)
F-Region triple splitting

G. R. ELLIS
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ABSTRACT
Measurements of the direction of arrival of Z echoes have been made at Hobart, Tasmania. The results indicate that F-region triple splitting is caused by back-scattering from a rough layer. The directions observed are consistent with the assumption that reflection at the Z level occurs when the angle of incidence is such that the wave normal becomes parallel to the geomagnetic field at the ordinary level of reflection.

INTRODUCTION
It is now well known that the Z component is polarized in the ordinary sense (Hogarth, 1951; Landmark, 1952). However, there is no general agreement on the mechanism responsible for F region triple splitting. ECKERSLEY (1950) and RYDEBECK (1950) have suggested that coupling between the O and Z waves at the ordinary level of reflection may be sufficient at vertical incidence to produce observable Z echoes. While there is some evidence that coupling in the E region may occur, this explanation requires a collision frequency at least ten times greater than the $10^4$ per second usually accepted as probable in the lower $F_2$ region.

SCOTT (1950) has advanced an alternative hypothesis that triple splitting may be due to longitudinal propagation of part of the incident wave which is reflected by a rough layer from an area centered on the magnetic zenith of the observer. In this case it would be expected that in the latitude of Hobart, Tasmania (geomagnetic latitude 51°S, Dip. 72°), Z echoes would be returned to the transmitter at an angle of at least 15° North of the vertical.

Measurements of the direction of arrival of Z echoes have been made recently at Hobart.

EXPERIMENTAL
The technique used was the well known one of observing the phase difference between the signals induced in two spaced loop aerials by a selected downcoming echo (Ross et al., 1951). A twin channel cathode ray direction-finding receiver was used to amplify the signals. The loop spacing was 120 feet.

It is considered that the combined site and lining-up zero error of the receiver did not exceed $\frac{1}{2}°$ for angles of arrival near vertical incidence. The error involved in measuring differences of angle did not exceed $0.2°$.

RESULTS
Measurements of the direction of arrival of F region Z echoes were made during July, September and October, 1952. The results obtained are summarised in Table 1.
The mean direction of arrival lay in the plane of the Geomagnetic field. In all cases the observations were made between 1500 and 1800 hours L.M.T. on a frequency slightly less than $f_s F_2$. It was found that these times gave the greatest probability of observing strong $Z$ echoes with a minimum of interference.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Mean angle of arrival</th>
<th>Number of Observations</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mc/s</td>
<td>Degrees North of Vertical</td>
<td></td>
<td>Degrees</td>
</tr>
<tr>
<td>4·65</td>
<td>8·9</td>
<td>220</td>
<td>0·55</td>
</tr>
<tr>
<td>5·2</td>
<td>8·75</td>
<td>42</td>
<td>0·4</td>
</tr>
</tbody>
</table>

Although triple splitting is frequently observed at Hobart in the $E$ and lower $F_1$ regions on $P'f$ records, these $Z$ echoes have been too weak for reliable direction finding.

The direction of arrival was observed to fluctuate rapidly over a limited range. Fig. 1 shows a typical example.

**Theoretical Considerations**

It is apparent that the $F$ region triple splitting observed is due to back scattering of the $Z$ wave by a rough layer as suggested by Scott (1950), although the angle
measured is considerably less than expected. In discussing the possible explanation the following symbols will be used:

\[ p = \text{angular wave frequency} \]
\[ N = \text{electron density} \]
\[ p_0^2 = \frac{4 \pi N e^2}{m} \]
\[ v = \text{collision frequency} \]
\[ \Theta = \text{propagation angle; the angle between the wave normal and the direction of the geomagnetic field} \]
\[ \Theta_0 = \frac{\pi}{2} - \text{angle of Dip} \]
\[ v_0 = \frac{p_H \sin^2 \Theta}{2 \cos \Theta} = \text{critical collision frequency} \]
\[ \Phi = \text{angle between wave normal and vertical} \]
\[ \Theta_0 = \text{propagation angle when } v = v_0 \]
\[ q = \text{APPLETON-HARTREE refractive index} \]

If a wave packet is vertically incident on a non-uniform layer the direction of the wave normal will remain vertical at all levels. Because of this the transition from transverse to longitudinal propagation is usually discussed with reference to curves of \( q^2 = q^2(N) \) for given values of \( \Theta \). Typical curves for zero collision frequency are shown in Fig. 2.

It is seen that the transition may be described in terms of changes in the shape of the curves at the transverse ordinary level, \( p_0^2 = p^2 \), as \( \Theta \to 0 \). In the \( Z \) region there is no qualitative difference between the transverse extraordinary mode and the longitudinal ordinary mode.

If a wave packet is obliquely incident on a non-uniform layer the direction of the wave normal will vary continuously as the wave penetrates the layer. We may conclude from the above discussion that if the angle of incidence is such that the wave normal of the ordinary wave tends to become parallel to the magnetic field as the wave packet approaches the ordinary level of reflection, then penetration of this level and reflection at the \( Z \) level may occur.

The variation in the direction of the wave normal may be obtained graphically by a method described by POEVERLEIN (1948). Fig. 3 shows a polar diagram of
the refractive index of the ordinary wave as a function of \( \Theta \) for values of \( p_0 < p^2 \). In Fig. 4 we have similar curves for propagation in the \( Z \) region where \( p_0^2 > p^2 \).

The angle of refraction of the wave normal \( \Phi \) for a given angle of incidence \( \Phi_i \) at any level is given by Snell's law

\[
\sin \Phi = \frac{q}{\sin \Phi_i}
\]

If then we draw a vertical line on the diagram at a distance \( \sin \Phi_i \) from the centre, then the directions of the wave normal at a given value of electron density are given by the radii at the intersections with the appropriate curve. Two directions are found, one for the upgoing and one for the downcoming wave.

It is seen from Fig. 3 that at the angle of incidence chosen the wave normal of the upgoing wave will become parallel to the field at the ordinary level of reflection. Continuing on Fig. 4, we have the same direction of wave normal at the same level. At higher levels \( \Phi \) increases and with a smooth layer the wave packet reflected at the \( Z \) level will arrive back at the ordinary level with its wave normal at a considerable angle to the field. It may or may not penetrate this level. If it did the observed polarisation of the \( Z \) echo would be extraordinary. In any case the wave packet would not return to the transmitting point. Only in the case of energy scattered at the \( Z \) level back along its incident path will the wave normal again become parallel to the field and longitudinal penetration at the ordinary level result.

To obtain the required angle of incidence \( \Phi_c \), it is only necessary to know the coordinates of the point \( P \). These are given by the inclination of the magnetic field \( \Theta_0 \) and the magnitude of the refractive index for longitudinal propagation when \( p_0^2 = p^2 \) and \( v = 0 \)

that is

\[
\sin \Phi_c = \sqrt{\frac{p_H}{p_H + p}} \sin \Theta_0
\]
F-Region triple splitting

Fig. 5 shows $\Phi_e$ as a function of frequency for different values of magnetic dip. The observed directions of arrival of $Z$ echoes at Hobart appear on the diagram.

**COLLISIONS**

According to the classical ray theory the effect of collisions is to increase from zero the propagation angle at which the transition from transverse to longitudinal propagation occurs. Corresponding to this range of $\Theta$ there will be a range of angles of incidence favouring a longitudinal penetration of the ordinary level. This is shown in Fig. 6.

Providing $\Theta_e$ is small we have

$$\Delta \Phi_e \sim \frac{2q \cos \Theta_o \cdot \Theta_e}{\cos \Phi_e}$$

If $\Theta_e$ is large as is the case for the $E$ region a more accurate calculation is necessary.

Typical values of $\Delta \Phi_e$ are given in Table 2.

The observed variation in the angle of arrival may be due therefore to collisions.

**BACK SCATTERING**

Since back scattering appears to be the primary cause of the observed triple splitting, it is relevant to examine the available evidence for roughness in the
reflecting layer. Unfortunately, only one series of observations have so far been reported. These were made by Briggs and Phillips (1950) in connection with an investigation of ionospheric winds. They have shown that the angular spreading of a downcoming wave reflected by a small element of the layer may be described by the function \( \cos^n \psi \) where \( \psi \) is the angle of scattering measured from the direction of specular reflection. They defined the roughness of the layer in terms of a parameter \( \psi_0 \) such that \( \cos^n \psi_0 = \frac{1}{4} \).

**Table 2. Variation of \( \Delta \Phi \) with frequency**

<table>
<thead>
<tr>
<th>( \Phi_0 = 18\degree )</th>
<th>( \psi = 10^4 ) per second</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \Phi )</td>
<td>( \frac{p}{p_0} )</td>
</tr>
<tr>
<td>Degrees</td>
<td>Degrees</td>
</tr>
<tr>
<td>2.55</td>
<td>1</td>
</tr>
<tr>
<td>3.05</td>
<td>2</td>
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<tr>
<td>2.05</td>
<td>3</td>
</tr>
<tr>
<td>2.0</td>
<td>4</td>
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</tbody>
</table>

During a series of observations at night on 2.4 Mc/s from January, 1949 to January, 1950, they found that \( \psi_0 \) did not exceed \( 18\degree \) for the \( F \) region. On 4.8 Mc/s between January and May, 1950, \( \psi_0 \) did not exceed \( 10\degree \).

From Fig. 5 we see that in Southern England (Dip 66\degree), spreading over an angle of \( 28\degree \) would be necessary at 2.4 Mc/s to return \( Z \) echoes to the transmitting point. Taking \( \psi_0 \) as \( 18\degree \) we have

\[
\cos^n \psi = 0.86
\]

that is, the power reflected back in the direction of incidence is one-fifth of that reflected specularly.

At 4.8 Mc/s and with \( \psi_0 = 10\degree \) we have

\[
\cos^n \psi = 10^{-0.6}
\]

We may conclude that during the periods mentioned triple splitting would have been most unlikely in Southern England on 4.8 Mc/s. It may have been observable on rare occasions at night on 2.4 Mc/s.

Since triple splitting occurs during the day time at Hobart on frequencies between 4.5 and 5.5 Mc/s it is apparent that the \( F \) layer at times must be considerably rougher here than has been observed in England.

\( \psi_0 \) would have to reach the vicinity of \( 18\degree \) compared with the maximum of \( 5\degree \) reported by Briggs and Phillips. The relation between \( F \) layer roughness and triple splitting at Hobart is being made the subject of a separate investigation.
**F-Region triple splitting**

**CRITICAL FREQUENCY OF THE Z WAVE**

As the Z wave is reflected at oblique incidence the critical frequency $f_c F_2$ observed on $P'f$ records will be greater than that calculated from the minimum ordinary wave critical frequency. The appearance of the $P'f$ record is shown in Fig. 7.

![Diagram](image)

Fig. 7. Position of Z trace in the presence of critical frequency spreading on the O and X traces.

Actual $P'f$ records showing F region triple splitting generally have this form.

**Acknowledgments**—This investigation has been made in association with the University of Tasmania which has provided facilities and equipment, aided by a grant from the Radio Research Board of the C.S.I.R.O.

**References**

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<th>Volume/Issue/Part</th>
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